Interaction of Conduction and Radiation in Anisotropically Scattering, Spherical Media

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Steady-state heat transfer by simultaneous conduction and radiation in absorbing, emitting, linear anisotropically scattering spherical media occupying the region $0 \le r \le R$ is analyzed. The medium, which contains continuous heat-generating sources, is confined within a gray, diffuse spherical boundary. To obtain a rigorous solution to the nonlinear heat-transfer problem, the integral form of the equation of transfer is utilized. This integral form is solved in a highly accurate manner by first expanding the unknown functions in power series and then employing the Galerkin method for generating a system of linear algebraic equations, which is then solved by standard methods. The energy equation for the temperature is discretized according to a standard finite-difference approach, and the resulting nonlinear system of equations is linearized utilizing the Newton-Raphson method and solved iteratively by standard routines. A comprehensive parametric study is performed in order to examine the interaction between the radiation and conduction.

Nomenclature

a_l	= expansion coefficients to $\theta^4(\tau)$
C_i	= expansion coefficients to $G^*(\tau)$
D_j	= expansion coefficients to $Q^r(\tau)$
f	= fractional scattering term
g'	= scattering asymmetry factor
$G(\tau)$	$= 2\pi \int_{-1}^{1} I(\tau,\mu)\, d\mu$, incident radiation
$G^*(\tau)$	$= G(\tau)/(4n^2\bar{\sigma}T_{\rm ref}^4)$, dimensionless incident radiation
$h(\tau)$	= dimensional source term
$H(\tau)$	$= h(\tau)R^2/(kT_{\rm ref})$, dimensionless source term
$I(\tau,\mu)$	= radiation intensity
k	= thermal conductivity
$L_{mn}^i(\tau)$	= function defined in Ref. 13
L_{mn}^{ij}	= function defined in Ref. 13
n	= index of refraction
N	$= (kT_{\rm ref}/R)/(4n^2\bar{\sigma}T_{\rm ref}^4)$, conduction-to-radiation parameter
$P(\mu,\mu')$	= scattering phase function, Eq. (3)
Q^c	$= -\tau_0 N(d\theta/d\tau)$ dimensionless conductive heat flux
$Q_n(\tau)$	= function defined by Eq. (9b) and evaluated using Eq. (12).
Q_n^i	= function defined by Eq. (15a)
$q^r(\tau)$	$= 2\pi \int_{-1}^{1} I(\tau,\mu)\mu\, d\mu$, radiation heat flux
$Q^r(\tau)$	$= q^r(\tau)/(4n^2\bar{\sigma}T_{\rm ref}^4)$, dimensionless radiation heat flux
Q^T	= dimensionless total heat flux
r	= radial variable
R	= radius of sphere
$T(\tau)$	= temperature
$T_{\rm ref}$	= reference temperature
T_w	= wall temperature
$U_n(\tau)$	= function defined in Ref. 13
U_n^i	= function defined in Ref. 13
$Y_n(\tau)$	= function defined by Eq. (9a)
Y_n^i	= function defined by Eq. (15b)
$\delta(\mu)$	= Dirac delta function
ϵ	= emissivity of wall surface
θ	$= T/T_{\rm ref}$, dimensionless medium temperature
θ_w	$= T_w/T_{\rm ref}$, dimensionless wall temperature
κ	= absorption coefficient
μ	= direction cosine
σ	= scattering coefficient
$\bar{\sigma}$	= Stefan-Boltzmann constant
τ	$= (1-\omega f)(\kappa+\sigma)r$, optical radial variable, Eq. (5a)
τ_0	= optical radius
χ^*	= function defined in Ref. 13
$\Psi(\tau,\mu)$	$= I(\tau,\mu)/(n^2\bar{\sigma}T_{\rm ref}^4/\pi)$, dimensionless radiation intensity
ω	$= \sigma/(\kappa+\sigma)$, single scattering albedo
ω_s	= scaled single scattering albedo, Eq. (5b)

Introduction

THE analysis of radiative transfer in participating media has received considerable attention in the literature. Much of the attention has been focused on one-dimensional plane-parallel systems, and an excellent review on the treatment of radiative transfer in various geometries has been given by Viskanta.[1] However, the analysis of radiative transfer in spherical media has been the subject of relatively few investigations, although there are important applications in numerous areas including, among others, spherical propulsion systems, nuclear energy generation and explosions, astrophysics, thermal insulation systems, pyrotechnics, and flares. Consequently, there is a need to continue the advancement of the investigation of radiative transfer and its interaction with other modes of energy transport.

Many of the previous investigations on radiative transfer in spherically symmetric geometry have been limited to the special case of an absorbing, emitting medium. Sparrow et al.[2] examined the effects of a heat-generating gas enclosed in a spherical shell. Viskanta and Lall[3] considered transient cooling of an absorbing, emitting sphere occupying the region $0 \le r \le R$ by thermal radiation. Rhyming[4] determined the temperature distribution between two black concentric spheres separated by an absorbing emitting gas with a constant absorption coefficient. Viskanta and Crosbie[5] extended the analysis of Rhyming[4] to include the effects of a uniformly heat-generating medium. Dennar and Sibulkin[6] evaluated the accuracy of the differential approximation for the solution to radiative transfer in a spherical shell containing a heat-generating, gray gas.

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There are several works that have included the effects of scattering by suspended particles within the medium. Viskanta and Merriam[7] analyzed steady-state heat transfer by both conduction and radiation in a heat-generating spherical shell containing absorbing, emitting, and isotropically scattering constituents. Tong and Swathi[8] used the spherical harmonics approximation to study the effects of anisotropic scattering on the temperature distribution in a nonconducting, uniformly heat-generating spherical shell. Recently, Tsai and Özişik[9] examined the time-dependent interaction of conduction and radiation in an absorbing, emitting, isotropically scattering sphere occupying the region $0 \le r \le R$. However, it appears that there are no studies available in the literature that have considered the effects of both conduction and radiation in an absorbing, emitting, anisotropically scattering sphere, although there are important applications of such investigations.

The objective of this work is to analyze the interaction between conduction and radiation in a sphere occupying the region $0 \le r \le R$, whose gray, diffuse bounding surface is maintained at a prescribed temperature. The gray constituents absorb, emit, and anisotropically scatter the thermal radiation, and the medium generate heat in prescribed manner. A rigorous solution to the nonlinear interaction problem is presented, and the numerical results of comprehensive parameter surveys are performed and discussed.

Analysis

Energy Equation

Consider one-dimensional, steady-state combined conduction and radiation heat transfer in an absorbing, emitting, anisotropically scattering sphere of radius R. A schematic of the physical model and coordinates is given in Fig. 1. The volumetric heat generation within the sphere is prescribed, and the bounding surface is maintained at a constant temperature. The mathematical formulation to this problem in dimensionless form is given by

$$\frac{d}{d\tau}\left[\tau^2\left(-\frac{d\theta(\tau)}{d\tau}+\frac{1}{N\tau_0}Q^r(\tau)\right)\right]=\left(\frac{\tau}{\tau_0}\right)^2 H(\tau) \quad \text{in} \quad 0<\tau<\tau_0 \tag{1}$$

and subject to the boundary conditions

$$\frac{d\theta(\tau)}{d\tau}=0, \qquad \tau=0 \tag{2a}$$

$$\theta(\tau_0)=\theta_w, \qquad \tau=\tau_0 \tag{2b}$$

where $\tau=\tau_0 r/R$ is a scaled optical radial variable.

Equation of Radiative Transfer

It is assumed that the radiation transfer takes place in an absorbing, emitting, gray, anisotropically scattering sphere occupying the region $0 \le r \le R$. The gray, diffuse bounding surface emits energy due its temperature. The scattering phase function is represented by the delta Eddington approximation.[10] This approximation includes an isotropic component, a linearly scattering component and a Dirac delta component, which removes much of the highly forward (or backward) scattering component of the scattering phase function typical to particles generated from the combustion of pulverized coal or in plumes of solid propellants. The delta Eddington scattering phase function is constructed as

$$P(\mu,\mu')=2f\delta(\mu-\mu')+(1-f)(1+3g'\mu\mu') \tag{3}$$

where f is the fractional scattering into the forward direction. Based on this scattering phase function, the $\Psi(\tau,\mu)$ satisfies the equation of radiative transfer given in the form[11]

$$\left[\mu\frac{\partial}{\partial\tau}+\frac{1-\mu^2}{\tau}\frac{\partial}{\partial\mu}+1\right]\Psi(\tau,\mu)$$

$$=(1-\omega_s)\theta^4(\tau)+\omega_s[G^*(\tau)+3g'\mu Q^r(\tau)]$$

$$-1\le\mu\le 1, \qquad 0<\tau<\tau_0 \tag{4a}$$

with a boundary condition of the form

$$\Psi(\tau_0,-\mu)=\epsilon\theta_w^4+2(1-\epsilon)\int_{\mu'=0}^{1}\Psi(\tau_0,\mu')\mu'\,d\mu', \quad \mu>0 \tag{4b}$$

where μ is the cosine of the angle between the direction of the radiation intensity and the scaled optical radial coordinate τ and ϵ is the emissivity of the bounding surface from the inside. Here, the τ and ω_s are, respectively, given by

$$\tau=(1-\omega f)(\kappa+\sigma)r \tag{5a}$$

$$\omega_s=[(1-f)\omega]/(1-\omega f) \tag{5b}$$

with

$$\omega=\sigma/(\sigma+\kappa) \tag{6}$$

being the single scattering albedo. Furthermore, an approach for determining f with $0\le f\le 1$ and g' with $-1\le g'\le 1$ using the actual scattering phase function is given in Ref. 10. Finally, $G^*(\tau)$ and $Q^r(\tau)$ are defined by

$$G^*(\tau)=\frac{1}{2}\int_{\mu=-1}^{1}\Psi(\tau,\mu)\,d\mu \tag{7a}$$

$$Q^r(\tau)=\frac{1}{2}\int_{\mu=-1}^{1}\Psi(\tau,\mu)\mu\,d\mu \tag{7b}$$

To solve the preceding problem, the integral form of the equation of transfer will be utilized as discussed next.

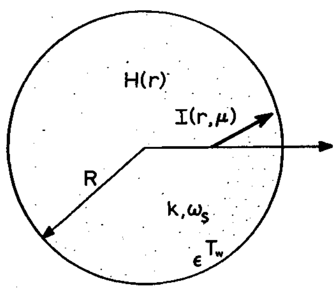

Fig. 1 The physical model and coordinates.

Integral Form of the Equation of Transfer

Equation (4a), subject to the boundary condition given by Eq. (4b), is transformable into a system of coupled Fredholm-type integral equations. As shown recently,[12] the corresponding system is represented by

$$\tau G^*(\tau)=\tau Y_0(\tau)+Q_0(\tau)+\frac{\omega_s}{2}\int_{x=0}^{\tau_0}[G^*(x)L_{00}(\tau,x)$$

$$+3g'Q^r(x)L_{10}(\tau,x)]x\,dx \tag{8a}$$

$$\tau Q^r(\tau)=\tau Y_1(\tau)+Q_1(\tau)+\frac{\omega_s}{2}\int_{x=0}^{\tau_0}[G^*(x)L_{01}(\tau,x)$$

$$+3g'Q^r(x)L_{11}(\tau,x)]x\,dx \tag{8b}$$

where

$$Y_n(\tau) = \frac{\epsilon}{2}\theta_w^4 \frac{\tau_0^2(-1)^n}{1-(1-\epsilon)\chi^*} U_n(\tau) \tag{9a}$$

$$Q_n(\tau) = \frac{1}{2}(1-\omega_s)\int_{x=0}^{\tau_0} \theta^4(x) L_{0n}(\tau,x) x \, \mathrm{d}x \tag{9b}$$

where x is the dummy variable of integration, and explicit expressions for the integrals $U_n(\tau)$, $n = 0,1$, the kernels $L_{mm}(\tau,x)$, $m,n = 0,1$ and constant χ^* have been presented[13] and are not included in this work.

Solution Method

To obtain a solution for the temperature, incident radiation, and net radiant heat flux, the following two difficulties of the proposed problem must be considered. First, the equation of transfer is a singular integro-differential equation for which a closed-form solution is not available; hence, an approximate solution to the radiation problem must be employed. Second, the coupling between the energy equation and the equation of radiative transfer is highly nonlinear; hence, an iterative solution scheme must be devised. The solution method for the radiation problem, which assumes that the temperature is "known," is presented next.

To solve the radiation problem, it is assumed that $H(\tau)$ is continuous and has continuous derivatives throughout the medium. First, we expand the functions $G^*(\tau)$, $Q^r(\tau)$, and $\theta^4(\tau)$ in finite power series as[13]

$$G^*(\tau) = \sum_{i=0}^{I} C_i \tau^i, \qquad 0 \le \tau \le \tau_0 \tag{10a}$$

$$Q^r(\tau) = \sum_{j=1}^{J} D_j \tau^j, \qquad 0 \le \tau \le \tau_0 \tag{10b}$$

$$\theta^4(\tau) = \sum_{\substack{l=0\\ l\neq 1}}^{L} a_l \tau^l, \qquad 0 \le \tau \le \tau_0 \tag{10c}$$

where C_i, $i = 0,1,\dots,I$, and $D_j, j = 1,2,\dots,J$, are the unknown expansion coefficients. Here, the power series for $Q^r(\tau)$ starts at $j = 1$, because

$$\lim_{\tau \to 0} q(\tau) = 0 \tag{11}$$

The expansion coefficients a_l, $l = 0,2,3,\dots,L$ are determined by using a standard least-squares fit, and by substituting Eq. (10c) into Eq. (9b) we obtain

$$Q_n(\tau) = \frac{1}{2}(1-\omega_s)\sum_{\substack{l=0\\ l\neq 1}}^{L} a_l L_{0n}^l(\tau) \tag{12}$$

where

$$L_{mn}^i(\tau) = \int_{x=0}^{\tau_0} L_{mn}(\tau,x) x^{i+1} \, \mathrm{d}x \tag{13}$$

Explicit expressions for the integrals $L_{mn}^i(\tau)$ have been presented elsewhere.[13] To solve for the unknown expansion coefficients, we substitute Eqs. (10) into Eqs. (8) and subsequently employ the Galerkin method, as outlined in Ref. 14. These operations establish a system of $I + J + 1$ linear algebraic equations of the form

$$\sum_{i=0}^{I} C_i \left[\frac{\tau_0^{i+k+2}}{i+k+2} - \frac{\omega_s}{2} L_{00}^{ik}\right] - 3g' \frac{\omega_s}{2} \sum_{j=1}^{J} D_j L_{10}^{jk}$$
$$= Y_0^k + Q_0^k, \qquad k = 0,1,\dots,I \tag{14a}$$

$$-\frac{\omega_s}{2}\sum_{i=0}^{I} C_i L_{01}^{il} + \sum_{j=1}^{J} D_j \left[\frac{\tau_0^{j+l+2}}{j+l+2} - 3g' \frac{\omega_s}{2} L_{11}^{jl}\right]$$
$$= Y_1^l + Q_1^l, \qquad l = 1,2,\dots,J \tag{14b}$$

where

$$Q_n^j = \frac{1}{2}(1-\omega_s)\sum_{\substack{l=0\\ l\neq 1}}^{L} a_l L_{0n}^{jl} \tag{15a}$$

$$Y_n^j = \frac{\epsilon}{2}\theta_w^4 \frac{\tau_0^2}{1-(1-\epsilon)\chi^*} U_n^{j-1} \tag{15b}$$

Here, explicit expressions for the integrals L_{mn}^{ij} and U_n^j have been developed.[13]

With the availability of the solution to the incident radiation and the net radiant heat flux anywhere in the medium, a solution to the energy equation for temperature can now be determined by a suitable method. Here, a standard finite-difference technique has been used. By replacing the divergence of the radiation heat flux by

$$\frac{1}{\tau^2}\frac{\mathrm{d}}{\mathrm{d}\tau}[\tau^2 Q^r(\tau)] = (1-\omega_s)[\theta^4(\tau) - G^*(\tau)] \tag{16}$$

and dividing the sphere into $M - 1$ layers, each having the thickness $\Delta\tau = \tau_0(M - 1)$, the resulting finite-difference equations become

$$\frac{6N\tau_0}{\Delta\tau^2}\theta_1 + (1-\omega_s)\theta_1^4 - \frac{6N\tau_0}{\Delta\tau^2}\theta_2$$
$$= (1-\omega_s)G_1^* + \frac{N}{\tau_0}H_1, \qquad m = 1 \tag{17a}$$

$$-\frac{N\tau_0}{\Delta\tau}\left(\frac{1}{\Delta\tau} - \frac{1}{\tau_m}\right)\theta_{m-1} + \frac{2N\tau_0}{\Delta\tau^2}\theta_m + (1-\omega_s)$$
$$\times \theta_m^4 - \frac{N\tau_0}{\Delta\tau}\left(\frac{1}{\Delta\tau} + \frac{1}{\tau_m}\right)\theta_{m+1}$$
$$= (1-\omega_s)G_m^* + (N/\tau_0)H_m, \qquad m = 2,\dots,M-2 \tag{17b}$$

$$-\frac{N\tau_0}{\Delta\tau}\left(\frac{1}{\Delta\tau} - \frac{1}{\tau_{M-1}}\right)\theta_{M-2} + \frac{2N\tau_0}{\Delta\tau^2}\theta_{M-1}$$
$$+ (1-\omega_s)\theta_{M-1}^4 = \frac{N\tau_0}{\Delta\tau}\left(\frac{1}{\Delta\tau} + \frac{1}{\tau_{M-1}}\right)\theta_w$$
$$+ (1-\omega_s)G_{M-1}^* + \frac{N}{\tau_0}H_{M-1}, \qquad m = M-1 \tag{17c}$$

where $\theta_m = \theta(\tau_m)$, $G_m^* = G^*(\tau_m)$, and $H_m = H(\tau_m)$, with $\tau_m = (m-1)\Delta\tau$. To solve for the temperatures θ_m, $m = 1,2,\dots,M-1$, Eqs. (17) are linearized and solved iteratively utilizing the Newton-Raphson method.[15]

Since the coupling between the energy equation and the equation of radiation transfer is highly nonlinear, it is necessary to employ iterative methods to solve the system of Eqs. (14) and (17). The iterative scheme is initiated by employing an expression for the temperature without the effects of radiative transfer. The least-squares method is subsequently used to generate a series expression for the temperature to the fourth power, which is substituted into Eqs. (14) for the solution to the expansion coefficients C_i, $i = 0,1,\dots,I$ and D_j, $j = 1,2,\dots,J$. This then yields a new approximation for the temperature. The iterative scheme continues until the relative change in the temperature at selected radial points between two consecutive iterations is less than 0.0001.

Once the temperature, incident radiation and net radiant heat flux are available, it is of importance to determine the contribution to the heat transfer by both conduction and radiation. First, the conductive flux is given by

$$Q^c(\tau) = \frac{q^c(\tau)}{4n^2\bar{\sigma}T_{\mathrm{ref}}^4} = -N\tau_0\frac{\mathrm{d}\theta}{\mathrm{d}\tau} \tag{18}$$

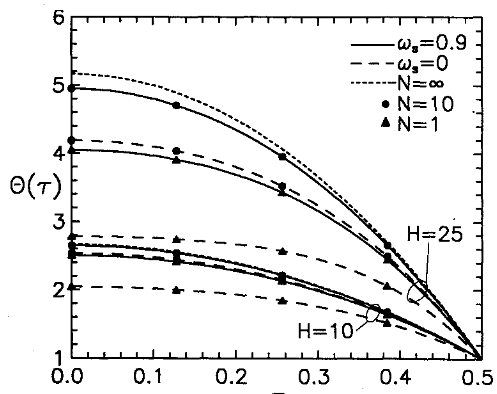

Fig. 2 The effects of scattering albedo ω_s, conduction-to-radiation parameter N, and heat generation H on the temperature distribution for large N; $\theta_w = 1$, $\epsilon_w = 1$, and $\tau_0 = 0.5$.

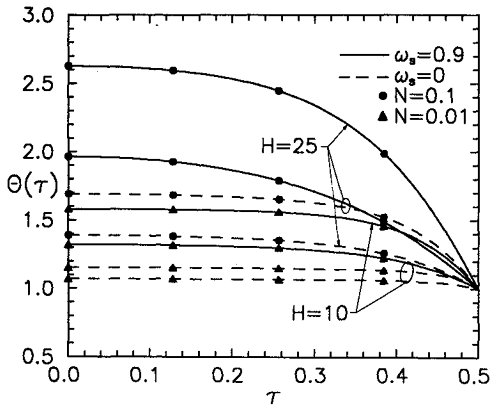

Fig. 3 The effects of scattering albedo ω_s, conduction-to-radiation parameter N, and heat generation H on the temperature distribution for small N; $\theta_w = 1$, $\epsilon_w = 1$, and $\tau_0 = 0.5$.

The dimensionless incident radiation and radiant heat flux are, respectively, most accurately computed from

$$\tau G^*(\tau) = \tau Y_0(\tau) + Q_0(\tau) + \frac{\omega_s}{2} \sum_{i=0}^{I} C_i L_{00}^i(\tau) + 3g' \frac{\omega_s}{2} \sum_{j=1}^{J} D_j L_{10}^j(\tau) \quad (19)$$

and

$$\tau Q^r(\tau) = \tau Y_1(\tau) + Q_1(\tau) + \frac{\omega_s}{2} \sum_{i=0}^{I} C_i L_{01}^i(\tau) + 3g' \frac{\omega_s}{2} \sum_{j=1}^{J} D_j L_{11}^j(\tau) \quad (20)$$

Finally, the total heat flux, defined as

$$Q^T(\tau) = Q^c(\tau) + Q^r(\tau) \quad (21)$$

can be obtained by integrating Eq. (1) and written as

$$Q^T(\tau) = \frac{N}{\tau^2 \tau_0} \int_{\tau'=0}^{\tau'=\tau} H(\tau')(\tau')^2 \, d\tau' \quad (22)$$

Discussion of Results

Inspection of Eqs. (1), (2), and (4) reveals that there are seven independent parameters: H, N, θ_w, τ_0, ω_s, g', and ϵ. To illustrate the interaction between the radiation and conduction heat transfer, a wide variety of independent solutions involving combinations of these parameters must be determined. The following presentation of results are based on a uniform dimensionless energy generation, $H(\tau) = H$, unit dimensionless wall temperature, $\theta_w = 1$, and a black bounding surface, $\epsilon_w = 1$. Since the sphere occupies the region $0 \le r \le R$ (or $0 \le \tau \le \tau_0$) and steady state exists, all the energy that is generated within the sphere must leave by both conduction and radiation at the boundary interface.

To illustrate the effects of the scattering for a smaller optical dimension, $\tau_0 = 0.5$, the steady-state temperature distribution within the sphere is shown in Figs. 2 and 3 for $N = \infty$, 10, 1 and $N = 0.1$, 0.01, respectively. The energy generation is selected as $H = 10$ and 25, and the radiative properties of the uniformly distributed particles are characterized by single scattering albedos $\omega_s = 0$ and 0.9. The ω_s represents the fraction of the radiant energy that is scattered from a pencil of rays, whereas $(1 - \omega_s)$ represents the remaining fraction that has been absorbed and transformed into thermal energy. Inspection of Figs. 2 and 3 reveals that scattering significantly increases the temperature distribution within the sphere even for such a small optical radius. For example, the center temperature is reduced from $\theta(0) \simeq 5.17$ in the pure conduction case ($N = \infty$) to $\theta(0) \simeq 4$ for the values of $N = 1$ and $\omega_s = 0.9$. Although the effects of anisotropically scattering is not shown in these figures ($g' = -0.9$ or $+0.9$), it should be pointed that such effects are

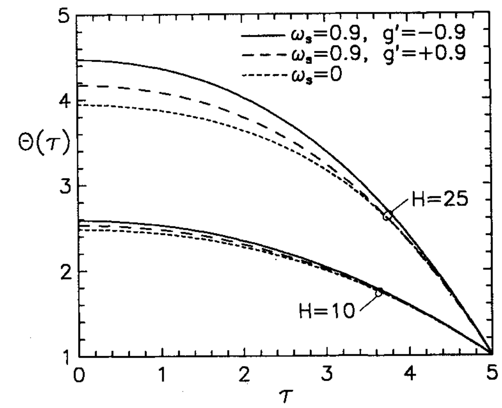

Fig. 4 The effects of scattering albedo ω_s, asymmetry factor g', and heat generation H on the temperature distribution; $N = 10$, $\theta_w = 1$, $\epsilon_w = 1$, and $\tau_0 = 5$.

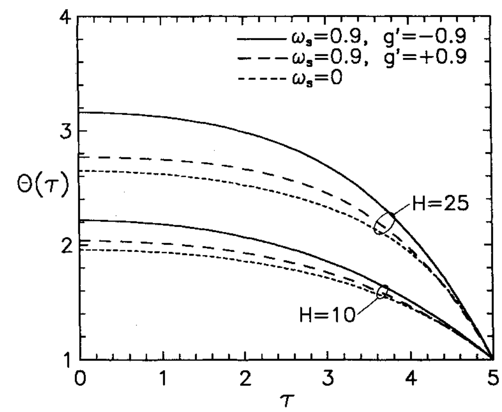

Fig. 5 The effects of scattering albedo ω_s, asymmetry factor g', and heat generation H on the temperature distribution; $N = 1$, $\theta_w = 1$, $\epsilon_w = 1$, and $\tau_0 = 5$.

insignificant; the results for the isotropically scattering case $\omega_s = 0.9$, $g' = 0$ are practically identical to the anisotropically scattering case of $\omega_s = 0.9$, $g' = -0.9$ or $+0.9$. Thus, the anisotropic scattering does not affect the predicted temperature distribution if the considered $\tau_0 \leq 0.5$.

Figures 4–6 show the temperature distributions in an optically thick sphere, $\tau_0 = 5$, for $N = 10$, 1, and 0.1, respectively. The uniform heat generation is chosen as $H = 10$ and 25, and the particles are characterized by $\omega_s = 0$ and 0.9, and asymmetry factors $g' = -0.9$ and $+0.9$. Examination of these figures reveals that the type of the scattering phase function has a significant influence on the temperature distribution; the core temperature is increased by more than 25% when the particles scatter significantly in the backward direction. Since the prescribed temperature boundary condition must be satisfied, the effects of the scattering on the temperatures near the bounding surface are smaller, but scattering influences appreciably the temperature gradient.

Figures 7 and 8 depict the fraction of the energy that leaves the sphere as radiation in the case of heat generation $H = 10$ and 25, respectively. Inspection of these figures in detail reveals that, if $N \ll 1$, then the radiation transfer in general accounts for most of the energy transfer at the bounding surface, but it is also significantly affected by the values of the optical radius and ω_s. Such effects can be anticipated by inspection of the dimensionless form of the energy equation, Eq. (1). For radiation-dominated situations, $N = 0.01$, cases a and b ($\omega_s = 0.9$, $g' = -0.9$ and $+0.9$) are the most sensitive to the amount of the heat generation, increasing from about 0.26 for $H = 10$ in Fig. 7 to about 0.33 for $H = 25$ in Fig. 8. For larger values of $N \gg 1$, the conduction accounts for most of the energy transfer at the bounding surface. Specifically, for $N = 10$ and the considered radiative properties, the radiation accounts for less than 3% of the total energy transfer from the sphere for $H = 10$ and about 8% for $H = 25$. Furthermore, a shielding effect[16] by the cold layer near the wall is clearly observed for the considered optical radius $\tau_0 = 5$ and $N > 0.1$. That is, if $\omega_s = 0$, the cold, nonscattering layer near the wall absorbs an appreciable portion of the radiant energy emerging from the interior of the sphere, whereas if $\omega_s = 0.9$, $g' = 0.9$, a larger fraction of the energy is either transmitted or sideways scattered or both to the bounding surface by the highly scattering particles. The shielding effect is not observed for the smaller optical radii $\tau_0 < 0.5$.

Finally, Figs. 9 and 10 show the effects of the conduction-to-radiation parameter on the temperature at the center of the sphere for heat generations $H = 10$ and 25, respectively. The single scattering albedos considered are $\omega_s = 0$ and 0.9, asymmetry factors $g' = -0.9$ and $+0.9$, and optical radii $\tau_0 = 0.05$, 0.5, and 5. Here, the line — — — — — identifies that the results for both $\tau_0 = 0.5$ (— — —) and $\tau_0 = 0.05$ (- - -) are practically the same. Examination of Fig. 9 reveals that, for strong radiation situations, $N \ll 1$, the center temperature is much lower than that of the pure conduction, which is $\theta(0) = 2.667$ for $N = \infty$. It is also noted that, for larger values of the conduction-to-radiation parameter, $N = 10$, the center temperature is reduced by approximately 10% for the larger values of the optical radii $\tau_0 = 5$. Inspection of Fig. 10,

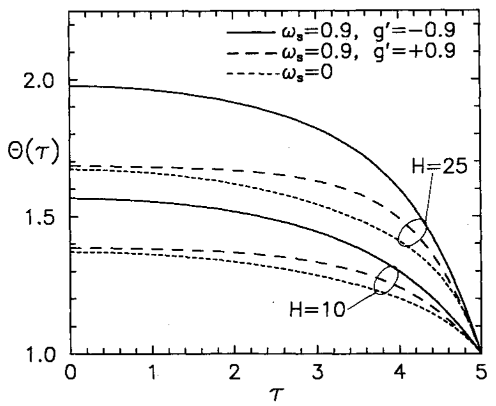

Fig. 6 The effects of scattering albedo ω_s, asymmetry factor g', and heat generation H on the temperature distribution; $N = 0.1$, $\theta_w = 1$, $\epsilon_w = 1$, and $\tau_0 = 5$.

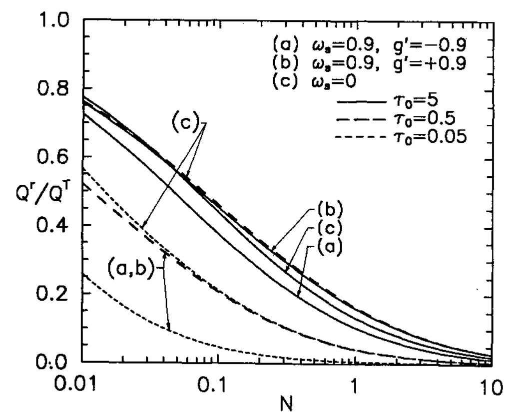

Fig. 7 The dependence of fraction of radiation flux at the bounding surface on the conduction-to-radiation parameter N; $H = 10$, $\theta_w = 1$, and $\epsilon_w = 1$.

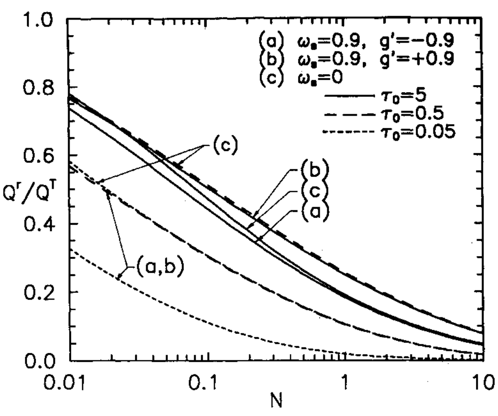

Fig. 8 The dependence of fraction of radiation flux at the bounding surface on the conduction-to-radiation parameter N; $H = 25$, $\theta_w = 1$, and $\epsilon_w = 1$.

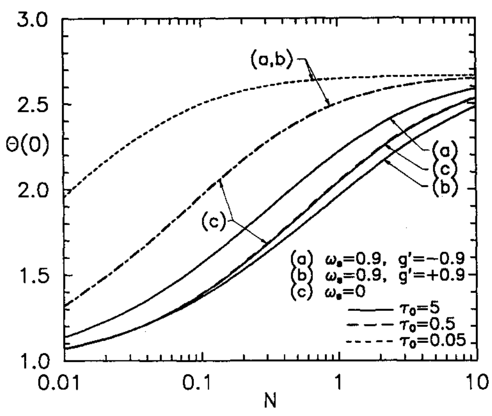

Fig. 9 The dependence of the centerline temperature $\theta(0)$ on the conduction-to-radiation parameter N; $H = 10$, $\theta_w = 1$, and $\epsilon_w = 1$.

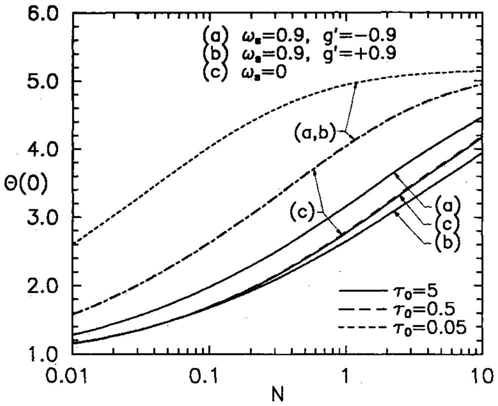

Fig. 10 The dependence of fraction of radiation flux at the bounding surface on the conduction-to-radiation parameter N; $H = 25$, $\theta_w = 1$, and $\epsilon_w = 1$.

however, shows that the center temperature is significantly reduced for all of the considered values of the N. For larger values of $N > 1$ and smaller optical radii $\tau_0 = 0.05$, the center temperature approaches the pure conduction case, which is $\theta(0) = 5.167$ for $n = \infty$. Observed in both Figs. 9 and 10 is the previously discussed shielding effect, which affects the center temperature only in the considered optically thick case of $\tau_0 = 5$ and for the larger values of conduction-to-radiation parameter $N > 0.1$.

The calculations of the previously discussed results have been performed on a VAX 8550, requiring only a few minutes of computer time for an $\tau_0 = 5$ to a few seconds for $\tau_0 = 0.05$. In this work, $I + 1 = J = L = 20$ and $M = 100$ have been employed for an $\tau_0 = 5$, $I + 1 = J = L = 10$ and $M = 40$ for $\tau_0 = 0.5$, and $I + 1 = J = L = 4$ and $M = 10$ for $\tau_0 = 0.05$. Since the required integrals are only functions of the τ_0, the solution scheme is very efficient for performing parameter surveys. In addition, it should also be noted that the energy equation, Eq. (1), can be integrated twice by utilizing the boundary conditions and the radiation flux expansion, Eq. (10b), to obtain an explicit expression for the temperature of the medium. This approach has been attempted, but it exhibited a much slower convergence than the standard finite-difference scheme given by Eq. (18).

Conclusions

An analysis of the interaction of heat conduction and radiation transfer in a steady-state heat generating, absorbing, emitting, anisotropically scattering sphere occupying the region $0 \le r \le R$ has been presented. The solution to the highly nonlinear problem has involved a standard finite-difference scheme for the energy equation and power series expansion techniques for the integral equations of radiative transfer. Based on the results presented in this work, which are similar to those found in the corresponding case of one-dimensional cylindrical geometry,[17] the following conclusions have been reached:

1) For optically thick spheres, $\tau_0 > 1$, the type of scattering by the uniformly suspended particles significantly influences the core temperatures; the presence of backward scattering particles always increases the core temperatures, whereas the presence of highly forward scattering particles reduces the core temperatures.

2) For intermediate and smaller optical radii ($\tau_0 \le 0.5$), the type of scattering may be neglected and simply represented as isotropic ($g' = 0$).

3) If $N > 0.1$ and larger optical radii ($\tau_0 = 5$), the radiative heat flux at the bounding surface experienced a slight enhancement for the highly forward scattering case of $\omega_s = 0.9$, $g' = +0.9$ compared to the nonscattering case of $\omega_s = 0$.

4) The center temperature $\theta(0)$ is reduced significantly from the pure conduction case even in the optically thin cases $\tau_0 < 0.5$ if $N < 0.1$.

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